Since $a^4 + a^2 + 1 = (a^2 + a + 1)(a^2 - a + 1)$, the claimed inequality is equivalent to

$$2(a-1)^2(a^2 + a + 1) \ \geq\ 0\,,$$

which is true, as $a^2 + a + 1 = (a + \frac{1}{2})^2 + \frac{3}{4}$ for all real numbers a.

3. The numbers p, $4p^2 + 1$, and $6p^2 + 1$ are primes. Determine p.

Solved by Arkady Alt, San Jose, CA, USA; Chip Curtis, Missouri Southern State University, Joplin, MO, USA; Geoffrey A. Kandall, Hamden, CT, USA; and Titu Zvonaru, Comăneşti, Romania. We give Alt's write-up.

First consider primes $p = 2$, 3, and 5.
If $p = 2$ then $4p^2 + 1 = 17$ is prime, but $6p^2 + 1 = 25$ is not prime.
If $p = 3$ then $4p^2 + 1 = 37$ is prime, but $6p^2 + 1 = 55$ is not prime.
If $p = 5$ then $4p^2 + 1 = 101$ and $6p^2 + 1 = 151$ are both primes.
Now let p be a prime greater than 5. Since

$$\begin{aligned} 4p^2 + 1 &= 5p^2 - (p^2 - 1) \ \equiv\ -(p^2 - 1) \pmod 5\,, \\ 6p^2 + 1 &= 5(p^2 - p - 1) + (p+2)(p+3) \\ &\equiv (p+2)(p+3) \pmod 5 \end{aligned}$$

and

$$-(p-1)p(p+1)(p+2)(p+3) \ \equiv\ 0 \pmod 5\,,$$

it follows that

$$p(4p^2 + 1)(6p^2 + 1) \ \equiv\ 0 \pmod 5\,.$$

Then $(4p^2+1)(6p^2+1) \equiv 0 \pmod 5$, because p and 5 are coprime. Hence, $4p^2 + 1$ or $6p^2 + 1$ is a composite number, because each is greater than 5 and one of them is divisible by 5.

Thus, the only solution to the problem is $p = 5$.

4. Prove that if two medians of a triangle are perpendicular, then the triangle whose sides are congruent to the medians of the original triangle is a right triangle.

Solved by Miguel Amengual Covas, Cala Figuera, Mallorca, Spain; Chip Curtis, Missouri Southern State University, Joplin, MO, USA; Geoffrey A. Kandall, Hamden, CT, USA; and Titu Zvonaru, Comăneşti, Romania. We give the two solutions by Amengual Covas.

First Solution: Let ABC be the given triangle, and let M, N, and P be the midpoints of the sides BC, CA, and AB, respectively. Let G be the centroid of $\triangle ABC$ and let D be symmetric to G with respect to M.